Dyadic Data Analysis

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Dyadic Component

1. Psychological rationale for homogeneity and interdependence
2. Statistical framework that incorporates homogeneity and interdependence
3. Give a few examples and develop intuition
Dyadic Component

1. Psychological rationale for homogeneity and interdependence
2. Statistical framework that incorporates homogeneity and interdependence
3. Give a few examples and develop intuition

Beginning, middle and end, but not necessarily in that order
Visualization Demonstration
Visualization Demonstration

Wife’s $X$ → $c$ ↓ $r_{XX'}$ → Husband’s $X$

a $X$ Error

b $c$ $Y$ Error

d $Y$ Error

Husband’s $Y$
Couples in similar stages of exercise change:

\[ R^2 = .44 \]

Patient Give \( \rightarrow \) Patient Receive

\[ a \times 0.62^* \]

Couples in different stages of exercise change:

\[ R^2 = .38 \]

Patient Give \( \rightarrow \) Patient Receive

\[ a \times 0.15 \]

Spouse Give \( \rightarrow \) Spouse Receive

\[ d \times 0.08 \]
Why Homogeneity and Interdependence?

How is an individual similar to or different from their spouse in thought, behavior or affect (e.g., shared norms)?

How do couple members influence each other?
Why Homogeneity and Interdependence?

How is an individual similar to or different from their spouse in thought, behavior or affect (e.g., shared norms)?

Similarity is operationalized by shared variance and correlated error

How do couple members influence each other?
Influence is operationalized by regression paths
Why Homogeneity and Interdependence?

How is an individual similar to or different from their spouse in thought, behavior or affect (e.g., shared norms)?

How do couple members influence each other?

We shouldn’t analyze data from dyads/groups as individuals.
They decided to repeat the wedding.
Interdependence

- We have good intuition and models for dealing with temporal dependence (time series, repeated measures, growth curves) and multivariate structure (factor and SEM models)
Interdependence

- We have good intuition and models for dealing with temporal dependence (time series, repeated measures, growth curves) and multivariate structure (factor and SEM models).
- We have weaker intuition about interdependence due to social interaction or pairing.
Interdependence

- We have good intuition and models for dealing with *temporal dependence* (time series, repeated measures, growth curves) and *multivariate structure* (factor and SEM models).
- We have weaker intuition about interdependence due to social interaction or pairing.
- We have good statistical models for each (e.g., HLM, SEM, latent growth curves), but lack a complete understanding of how these frameworks interrelate.
Desiderata for a General Framework

Provide a conceptual framework for representing structure in data due to time, grouping, and multiple variables.
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Easily handle covariates and common procedures such as mediation and moderation.
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Easy to use with standard designs but flexible to deal with nonstandard design elements
Nonindependence

Correlations due to
Nonindependence

Correlations due to

• temporal clustering
Nonindependence

Correlations due to

- temporal clustering
- variable clustering
Nonindependence

Correlations due to

- temporal clustering
- variable clustering
- interpersonal clustering
### Independence

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Independence as the Null Hypothesis

Example with discrete behavior (group size $= n$):

1. Assume the behavior can be represented as independent, identically distributed Bernoulli trials (i.e., coin flips).
2. Estimate the probability $p$ that the behavior occurs when individuals are alone.
3. Calculate the null hypothesis of independence for groups of size $n$ as $1 - (1 - p)^n$. This is the estimated probability that at least one member of the group will exhibit the behavior.
4. Compare the observed proportion of the behavior in groups of size $n$ to the null value computed in the previous step.

Limitation: interdependence is not a direct parameter!
Independence as the Null Hypothesis

Example with discrete behavior (group size = $n$):

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Independence as the Null Hypothesis

Example with discrete behavior (group size = \( n \)):

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Independence as the Null Hypothesis

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4. Compare the observed proportion of the behavior in groups of size $n$ to the null value computed in the previous step.

Limitation: interdependence is not a direct parameter!
Intraclass Correlation

The intraclass correlation will have the leading role in this play. We’ll denote it as

\[ r_{xx'} \]
Intraclass Correlation & Waldo

$r_{xx'}$
Intraclass Correlation

ANOVA/HLM Language: Two level model approach

\[ Y_{ij} = \beta_i + \epsilon_{ij} \]
\[ \beta_i = \mu + \pi_i \]
Intraclass Correlation

ANOVA/HLM Language: Two level model approach

\[ Y_{ij} = \beta_i + \epsilon_{ij} \]
\[ \beta_i = \mu + \pi_i \]

Intraclass correlation is given by

\[ r_{xx'} = \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_\epsilon} \]
Intraclass Correlation

ANOVA/HLM Language: Two level model approach

\[ Y_{ij} = \beta_i + \epsilon_{ij} \]
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Intraclass correlation is given by

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A proportion interpretation.
Intraclass Correlation: Another approach

ANOVA/HLM Language: Two level model

\[ Y_{ij} = \beta_i + \epsilon_{ij} \]
\[ \beta_i = \mu + \pi_i \]

Intraclass correlation is given by

\[ r_{xx'} = \frac{\sigma^2_\pi}{\sigma^2_\pi + \sigma^2_\epsilon} \]
The Nested Individual
Symbolic representation for the pairwise setup

The first subscript represents the dyad and the second subscript represents the individual.

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Concrete Illustration of the Pairwise Coding

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Pairwise Plots

strangers; \( r_{xx'} = 0.72 \)

friends; \( r_{xx'} = 0.4 \)
Pairwise Intraclass for the Exchangeable Case

Simply compute the usual Pearson correlation between variable X and the “reverse coded” version of X, which we denote $X'$. The significance test (against a null hypothesis of zero) is simply

$$Z = r_{xx'} \sqrt{n}$$

where $Z$ is asymptotically normally distributed and $n$ is the number of dyads.
Pairwise Plots

not on welfare

welfare
Symbolic representation for the pairwise setup

The first subscript represents the dyad and the second subscript represents the individual. Categorization of individuals as 1 or 2 is based on the class variable C.

<table>
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Pairwise Intraclass for the Distinguishable Case

Compute the partial correlation between variable X and the “reverse coded” version of X, partialling out the person code C.

The partial pairwise intraclass correlation is given by

\[
r_{xx'.c} = \frac{r_{xx'} - r_{cx}r_{cx'}}{\sqrt{(1 - r_{cx}^2)(1 - r_{cx'}^2)}}
\]
Intraclass Correlation

The structural model underlying the intraclass correlation for the *exchangeable* case is

\[ Y_{ij} = \mu + \pi_i + \epsilon_{ij} \]

where \( \pi \) is a random effect. The parameter \( \pi \) represents the “dyad effect.” This model is equivalent to a one-way random-effects ANOVA with “dyad” as the factor.

The structural model for the *distinguishable* case is

\[ Y_{ijk} = \mu + \pi_i + \alpha_j + \epsilon_{ijk} \]

where \( \pi \) is a random effect and \( \alpha \) is a fixed effect. The parameter \( \pi \) represents the “dyad effect” and the parameter \( \alpha \) represents the effect on the “distinguishable” variable. This model is equivalent to a two-way ANOVA with “dyad” as a random-effects factor. The intraclass correlation in the distinguishable case will be numerically similar to the Pearson correlation in most situations.
The standard definition of the intraclass correlation is

$$
\rho_I = \frac{\text{MSB} - \text{MSE}}{\text{MSB} + (k - 1)\text{MSE}}
$$

The terms MSB and MSE come from the ANOVA source table, and $k$ represents the number of people in the “group” (i.e., in dyads $k = 2$). The same formula is used whether a one-way ANOVA (exchangeable case) or a two-way ANOVA (distinguishable case) is used. The intraclass correlation compares the variability between dyads v. the variability within dyads.
But the ANOVA approach is difficult to work with...

1. tedious to generalize to situations with many variables
2. not easy to develop intuition for the relevant mean square terms and to connect the parameters to meaningful psychological statements.
3. not easy to develop tests of significance

The ANOVA approach can be generalized through “hierarchical linear models” (HLM).

The **pairwise approach** is a special case of HLM when all groups have the same size (as in dyads), i.e., in the case of dyads the pairwise approach is identical to HLM. The main benefit of the pairwise approach is that it is easy to understand and provides natural connections with psychological research questions.
The pairwise intraclass is similar to the ANOVA intraclass but it is based on sums of squares rather than mean squares. For the special case of dyads we have

$$
\rho_p = \frac{SSB - SSE}{SSB + SSE} \tag{1}
$$
**SPSS**

MIXED dv BY person  
   /fixed person  
   /print solution testcov  
   /repeated = person | SUBJECT(dyad) covtype(CS).

---

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Wald Z</th>
<th>Sig.</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Repeated Measures: CS covariance</td>
<td>401.511364</td>
<td>174.276433</td>
<td>2.304</td>
<td>.021</td>
<td>(59.935832, 743.086895)</td>
</tr>
</tbody>
</table>

a. Dependent Variable: score.
Dyadic Correlation Between Two Variables

Example: Each member of a couple completes both a trust scale (e.g., how much do you trust your partner) and a satisfaction scale (e.g., how satisfied are you with your marriage).

What is the relationship between trust and satisfaction?
How would you approach this analysis problem?

1. correlate the trust scores with the satisfaction scores ignoring group membership
2. correlate mean trust score (within couple) with mean satisfaction score
How would you approach this analysis problem?

1. correlate the trust scores with the satisfaction scores ignoring group membership
2. correlate mean trust score (within couple) with mean satisfaction score

There are problems with these two correlations!

The first confounds dyad-level effects and the second confounds individual-level effects.

Thus, these two correlations are indeterminate as to the “psychological” mechanisms they represent.
Symbolic representation for the pairwise setup for two variables

<table>
<thead>
<tr>
<th>Dyad #</th>
<th>C</th>
<th>X</th>
<th>X'</th>
<th>Y</th>
<th>Y'</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>X_{11}</td>
<td>X_{12}</td>
<td>Y_{11}</td>
<td>Y_{12}</td>
</tr>
<tr>
<td>2</td>
<td>X_{12}</td>
<td>X_{11}</td>
<td>Y_{12}</td>
<td>Y_{11}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>X_{21}</td>
<td>X_{22}</td>
<td>Y_{21}</td>
<td>Y_{22}</td>
</tr>
<tr>
<td>2</td>
<td>X_{22}</td>
<td>X_{21}</td>
<td>Y_{22}</td>
<td>Y_{21}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X_{31}</td>
<td>X_{32}</td>
<td>Y_{31}</td>
<td>Y_{32}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X_{32}</td>
<td>X_{31}</td>
<td>Y_{32}</td>
<td>Y_{31}</td>
<td></td>
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<tr>
<td>2</td>
<td>X_{41}</td>
<td>X_{42}</td>
<td>Y_{41}</td>
<td>Y_{42}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>X_{42}</td>
<td>X_{41}</td>
<td>Y_{42}</td>
<td>Y_{41}</td>
<td></td>
</tr>
<tr>
<td>Dyad</td>
<td>Person</td>
<td>Var</td>
<td>y1</td>
<td>y2</td>
<td>y3</td>
</tr>
<tr>
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<td>3</td>
<td>2</td>
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</tr>
</tbody>
</table>
Graphical Representation of the Correlations
According to the model, the two observed correlations decompose as

\[ r_{xy} = \sqrt{r_{xx}'} r_d \sqrt{r_{yy}'} + \sqrt{1 - r_{xx}'} r_i \sqrt{1 - r_{yy}'} \]

and

\[ r_{xy'} = \sqrt{r_{xx}'} r_d \sqrt{r_{yy}'} . \]
According to the model, the two observed correlations decompose as

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and

\[ r_{xy}' = \sqrt{r_{xx}'} r_d \sqrt{r_{yy}'} . \]

With these decompositions, simple algebra solves for \( r_i \) and \( r_d \).

\[ r_i = \frac{r_{xy} - r_{xy}'}{\sqrt{1 - r_{xx}'} \sqrt{1 - r_{yy}'}} \]

and

\[ r_d = \frac{r_{xy}'}{\sqrt{r_{xx}'} \sqrt{r_{yy}'}} . \]
Example: Individual and Dyad Level Relationship

Frequency of verbalization and frequency of gaze.

<table>
<thead>
<tr>
<th></th>
<th>$r_i$</th>
<th>$r_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strangers</td>
<td>-.33</td>
<td>.68</td>
</tr>
<tr>
<td>Friends</td>
<td>.14</td>
<td>.30</td>
</tr>
</tbody>
</table>
SAS

proc calis cov edf=N-1 se method=mls residual pcorr;
lineqs
v1 = 1 F1 + E1,
v2 = 1 F1 + E2,
v3 = 1 F2 + E3,
v4 = 1 F2 + E4;

STD
F1-F2 = v1 v2,
E1-E4 = x1 x1 x2 x2;

COV
F1 F2 = rd,
E1 E3 = ri,
E2 E4 = ri;
run;

Cov matrix as input; state N.
Title: SEM model;
Data: File = G:\FTS\files from Rich\SEM data L3.dat;
variable: names = ID x1 x2 y1 y2;
USEV = x1 x2 y1 y2;
Analysis: type = meanstructure;
model:
  x by x1@1 x2@1;
  y by y1@1 y2@1;
  x1 x2 (1);
  y1 y2 (2);
  [x1 x2] (4);
  [y1 y2] (5);
  x1 with y1 (3);
  x2 with y2 (3);
  x with y;

output: sampstat standardized;
Correlation Between Dyad Means

\[ r_m = \frac{r_{xy} + r_{xy'}}{\sqrt{1 + r_{xx'}} \sqrt{1 + r_{yy'}}} \]

Note that \( r_m \) can be positive under different combinations of \( r_{xy} \) and \( r_{xy'} \). That is, \( r_m \) reflects a combination of individual and dyad level processes, and should not be routinely interpreted as reflecting only dyad level processes.
Latent Variable Model: HLM Lingo

Three-level model: one level for the variable, one level for individual effect, and one level for group effect.

\[ Y_{ijk} = \beta_0 X_0 + \beta_1 X_1 \]
\[ \beta_0 = \mu_0 + \pi_0 + \epsilon_0 \]
\[ \beta_1 = \mu_1 + \pi_1 + \epsilon_1 \]

\[ \pi \sim N \left(0, \begin{bmatrix} V_{\pi_0} & C_{\pi_0\pi_1} \\ C_{\pi_0\pi_1} & V_{\pi_1} \end{bmatrix} \right) \]
\[ \epsilon \sim N \left(0, \begin{bmatrix} V_{\epsilon_0} & C_{\epsilon_0\epsilon_1} \\ C_{\epsilon_0\epsilon_1} & V_{\epsilon_1} \end{bmatrix} \right) \]
Alternative Model: Interdependence

The degree to which one individual influences another (e.g., Lewin).

This influence need to occur face-to-face:

We have a good time together, even when we’re not together. Yogi Berra
Kelley & Thibaut: Early APIM

Interaction separated into three types of control or influence

1. actor effect (reflexive)
2. partner effect (fate)
3. mutual effect (behavior)
APIM is a pairwise model

\[ Y = \beta_0 + \beta_1 X + \beta_2 X' + \beta_3 XX' \]

such that

- predictor X represents the actor’s influence on the actor’s Y,
- predictor X’ represents the partner’s influence on the actor’s Y,
- the product XX’ represents the mutual influence of both people on the actor’s Y.
Example: Generalized Pairwise Model

Regress frequency of smiles/laughter on frequency of verbalization

Strangers: an effect of the partner’s verbalization frequency on the actor’s laughter (i.e., in ordinal language, the more the other talks, the more the actor smiles); no other effects
Example: Generalized Pairwise Model

Regress frequency of smiles/laughter on frequency of verbalization

Strangers: an effect of the partner’s verbalization frequency on the actor’s laughter (i.e., in ordinal language, the more the other talks, the more the actor smiles); no other effects

Friends: an effect of the actor’s verbalization frequency on the actor’s laughter (i.e., in ordinal language, the more I talk, the more I smile); no other effects
Simple Structural Models for Dyadic Designs (Modeling Partner Effects)

Basic Actor-Partner Model:
- Kraemer-Jacklin model of sex differences
- Nonstandard regression because of interdependence across cases
- Subject is unit of analysis and each dyad is “double-coded”

Generalized Actor-Partner Model: Simple generalization to continuous predictors
- Stinson and Ickes example
Actor-Partner Model Across Sex: Distinguishable version

- Couples is unit of analysis
- Individual relations within couples modeled by Structural Equation Modeling
- Test whether distinguishable paths are necessary (sex differences in processes)
- Murray, Holmes, & Griffin example
Actor-Partner Model with Heterosexual Married Couples

Actor Effect (a & d): .32
Partner Effect (b & c): .30
Actor-partner model: The ICC again

\[ Y = \beta_0 + \beta_1 X + \beta_2 X' \]

such that

- actor’s X predicts actor’s Y and
- partners X (denoted X’) predicts actor’s Y
Actor-partner model: The ICC again

\[ Y = \beta_0 + \beta_1 X + \beta_2 X' \]

such that

- actor’s X predicts actor’s Y and
- partners X (denoted X’) predicts actor’s Y

The actor regression coefficient can be expressed in terms of pairwise correlations

\[ \beta_1 = \frac{s_y(r_{xy} - r_{xy'}r_{xx'})}{s_x(1 - r_{xx'}^2)} \]

Similarly, the partner regression coefficient is

\[ \beta_2 = \frac{s_y(r_{xy'} - r_{xy}r_{xx'})}{s_x(1 - r_{xx'}^2)} \]
Variance of $\beta$ related to the ICC

The variance of the actor $\beta$ is

$$V(\beta_{\text{actor}}) = \frac{s_y^2(r_{xy'}^2r_{xx'}^2 - r_{xx'}r_{yy'} + 1 - r_{xy'}^2)}{2Ns_x^2(1 - r_{xx'}^2)}$$
Longitudinal Models

Get complicated. Different ways of representing change in a single person, now there are two individuals.
McArdle's Bivariate Latent Difference Model
across time are nested within individuals. Put another way, occasion is the Level 1 or lower-level unit and person is the Level 2 or upper-level unit. Conceptually, a multilevel analysis begins by stating that the data is hierarchical in nature and that the parameters of interest may vary across different levels.

The sample as follows: the lower level intercepts and slopes are aggregated across the four levels of time and time of measurement serves as the independent variable. This gives the following lower level equation:

$$\text{Conflict}_{ij} = \alpha_0 + \alpha_1 T_{ij} + e_{ij}$$

where $\alpha_0$ is the intercept, $\alpha_1$ is the slope, $T_{ij}$ is the time of measurement (taking on values 0, 2, 4, and 6) for child $i$ at age 11. The variance for the slope is based on the variance of the error represents the part of child $i$'s initial conflict score, and the slope, $b_1$, is predicted by the time variable, which is assumed to be 0 for the age 11 observation.

These models result in two fixed-effect estimates: an average intercept (i.e., the average conflict score at age 11 when time is treated as an age slope, a variance for the intercept and a variance for the slope, and a slope variable. To identify the intercept variable, we fixed the unstandardized path coefficient to the age 11 observation to 0. This defines the intercept variable as the value of conflict at age 11. The remaining unstandardized slope paths to the age 13, 15, and 17 observations are fixed standardized slope paths to the age 13, 15, and 17 observations are fixed standardized slope paths to the age 13, 15, and 17 observations are fixed.

In addition to the usual assumption that the errors have zero means, researchers who estimate SEM analyses. In this model, the four observations of conflict are treated as indicators of two latent variables, an intercept variable and a slope variable. To identify the slope variable, we fixed the unstandardized path coefficient to the age 11 observation to 0. This defines the intercept variable as the value of conflict at age 11. The remaining unstandardized slope paths to the age 13, 15, and 17 observations are fixed. As can be seen in Figure 1, the growth model in the SEM framework includes the same basic parameters as the growth model in the MLM framework: an average intercept and an average slope; $e_0, \text{var}(e_1), e_2, \text{var}(e_2), e_3, \text{var}(e_3), e_4, \text{var}(e_4)$.
Shared Variance Intercept/Slope

\[ \begin{align*}
&\text{E11} \rightarrow \text{TA Conflict 11} \\
&\text{E21} \rightarrow \text{TA Conflict 14} \\
&\text{E31} \rightarrow \text{TA Conflict 17} \\
&\text{E12} \rightarrow \text{TB Conflict 11} \\
&\text{E22} \rightarrow \text{TB Conflict 14} \\
&\text{E32} \rightarrow \text{TB Conflict 17}
\end{align*} \]
Which Programs to Use?

Multilevel models provide a unified approach to dyadic and longitudinal models.

**Advantages:** Arbitrary nested models with multiple levels of analysis

**Disadvantages:** Methods complex to implement and interpret
Which Programs to Use?

SEM provides a unified approach to dyadic and longitudinal models.

**Advantages:** Multiple variables easy to handle

**Disadvantages:** Difficult to implement unequal size groups; longitudinal designs can get complicated
For Now...

- Single framework for many different models
- Simple intuition for special case models (e.g., show correlations imposed by various models)
- Allows one to mix and match elements from different frameworks, including categorical dependent variables
- Allows extensions of standard models such as an individual being a member of multiple groups, an individual or family belonging to multiple neighborhoods, an individual belonging to multiple dyads

But we need a better program that makes it easy to fill in the correlated error structure.
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But we need a better program that makes it easy to fill in the correlated error structure
Conclusions

- Dependence due to social interaction does not require a “statistical cure”
- Interdependence provides an opportunity to measure and model social interaction (even over time)
- Ask “how can I capture the dependencies that are logically possible in my data”
Prescriptions

• The idea is not to “cure” non-independence but to study and conceptualize interdependence.
• Follow your conceptual models in their richness.
• There is still lots of room for careful design in longitudinal correlational research with dyads. It is not just a statistical issue.
Interdependence Mantra

Study
Model
Celebrate
Interdependence