



Stage Machine Design Competition 2023: Guess Who?

# Project Proposal

**Prepared by:**

Hanna Bowen

Matt Grenier

Jesse McKenzie

Raven Shales

**Advisor:** Bobby Reynolds

**Representing:**

University of Illinois Urbana-Champaign

Theatre Department Scenic Technology Program

Level 21 at Krannert Center for the Performing Arts

**February 28, 2023**

# Table of Contents

<b>The Team</b>	<b>2</b>
Team Schedule	2
Team Resources	2
<b>The Challenge</b>	<b>3</b>
<b>Concept Generation</b>	<b>5</b>
Brainstorming	5
Research	6
Concept Sketches	8
Calculations	9
<b>Concept Selection</b>	<b>11</b>
The Concept	11
Functional Decomposition	12
Assessment	13
<b>Design Specification</b>	<b>14</b>
Next Steps	15
Detailed Design	15
Build and Testing	15
<b>Appendix A: Concept Sketches</b>	<b>16</b>
<b>Appendix B: Research</b>	<b>22</b>
<b>Appendix C: Calculations</b>	<b>26</b>

## The Team

*Alma Otter Designs* is a team formed by three undergraduate students and one graduate student, all having a concentration in Scenic Technology at the University of Illinois Urbana-Champaign College of Fine and Applied Arts. Through a shared interest in theatrical design and fabrication, the team was formed to compete in the 2023 Stage Machine Design Competition.



The team's advisor, Bobby Reynolds, is the Scene Shop Supervisor at the Krannert Center for the Performing Arts and an alum of the Scenic Technology program. Inspiring the creation of the team, Bobby also competed in the Stage Machine Design Competition when he was a graduate student.

## Team Schedule

The standing objective of the team is to meet as a group for a minimum of 2 hours each week, which began with a kick-off meeting on January 16th. Below is the team's project schedule leading up to the competition event on May 6th.

- Research and Concept Generation: January 16 - February 5
- Concept Evaluation and Selection: February 6 - February 19
- Proposal Authoring: February 20 - February 27
- Detailed Design: February 28 - March 26
- Build and Testing: March 27 - April 24
- Final Submission Preparation: April 24 - May 5

## Team Resources

**Manufacturing:** The team will have access to the Krannert Center for the Performing Arts scene shop, which includes metal, carpentry, and fine-woodworking tools. The shop also has a 4x8 *ShopBot* CNC router that may be useful for fabrication of machine components. The team also has access to the maker space at Siebel Center, a design and research collaboration space that includes laser and waterjet cutting capabilities.

**Materials:** While there is no allocated budget to purchase materials for this project, the team has access to scrap shop materials and a large inventory of automation components in various onsite and offsite storage facilities. Old student projects also serve as sources of critical components. Sustainability through the repurposing of old equipment is a major consideration in the team's ultimate design and build.

## The Challenge

Per the SMDC23 challenge description: *“You and your team have been hired to design a machine for an upcoming museum event centered around family game night. Your specific challenge: for an installation based on the game Guess Who, children will have to be able to flip panels that are approximately 3’ wide by 5’ long. The event designers have created the actual flipping panels and the structures they are housed in; your task is to design a drive train that would allow a child to flip these panels by hand, at a distance from the axis of rotation.”*

*“The museum will provide the rotating panel (including the base it is installed into). You will need to design a mechanism that allows a participant to push on a handle to cause the panel to pivot up and to pull on that handle to cause the panel to pivot back down. Participants will vary in age from very young (around five years old) to parents and other adults; consequently, the range of motion of the handle and the force required to move it must be sized appropriately. Depending on the testing set up, adults, on average, have been found to be able to push with anywhere between 70 N (16 lb) and 200 N (45 lb), whereas children around age five—depending on the testing set up—have been found to be able to push with between 1 N (.25 lb) and 75 N (16lb) of force. For the purposes of this display, the museum staff has settled on a maximum push force of approximately 22 N (5 lb) of force. Similarly, the average length of a five-year-old’s arm is approximately 8 to 12 inches; consequently, the museum staff would like the pushing/pulling action required for this effect to be no more than the minimum length: 8 inches. Additionally, the operating height of the lever/handle should be 21 inches from the floor.”*


Per the notes in the drawings on the following page, it is further understood that the lever/handle does not need to travel in a straight line; it can rotate like a crank as long as the maximum horizontal translation is 8 inches. Therefore, a round handwheel can be used as long as its diameter does not exceed 8 inches. Additionally, the panel will have a weight of approximately 28 lbs and must remain in its fully lowered and raised positions without any additional force being applied by the operator.

**ROTATING PANEL - OPERATION**

**NOTE**

Your design will need to fulfill the following expectations:

- Transfer a maximum applied force of 22 N (5 lb) at the handlebar to the shaft of the provided rotating panel with sufficient space to pivot the panel.
- Transfer the horizontal translation of the handlebar of 8" to the shaft of the provided rotating panel such that the panel rotates through the selected 100° of travel. (Note: the handle bar will not need to travel only in a straight line-it can rotate around some axis as a result of team joint operation. The maximum horizontal translation must be no more than 8".)
- Transfer the motion of the handle to the shaft of the provided rotating panel such that a push on the handle will cause the panel to pivot up and a pull will cause the panel to pivot back down.
- The rotating panel should remain in place without any additional force applied at the end of travel in each direction.
- The interface point to the rotating panel will comprise a 1" diameter tapered shaft. It will not be provided; you will need to ensure that your mechanism can be securely attached to the shaft for operation.
- The rotating panel will be constructed using 3/4" plywood for the given size (120") it will weigh approximately 20 lb.



CHILDREN'S MUSEUM	
DATE SUBMITTED:	2
REVISED:	
TEAM NAME:	


**ROTATING PANEL - TOP VIEW**

**ROTATING PANEL - SIDE VIEW**

**NOTE**

Your design will need to fulfill the following expectations:

- Transfer a maximum applied force of 22 N (5 lb) at the handlebar to the shaft of the provided rotating panel with sufficient space to pivot the panel.
- Transfer the horizontal translation of the handlebar of 8" to the shaft of the provided rotating panel such that the panel rotates through the selected 100° of travel. (Note: the handle bar will not need to travel only in a straight line-it can rotate around some axis as a result of team joint operation. The maximum horizontal translation must be no more than 8".)
- Transfer the motion of the handle to the shaft of the provided rotating panel such that a push on the handle will cause the panel to pivot up and a pull will cause the panel to pivot back down.
- The rotating panel should remain in place without any additional force applied at the end of travel in each direction.
- The interface point to the rotating panel will comprise a 1" diameter tapered shaft. It will not be provided; you will need to ensure that your mechanism can be securely attached to the shaft for operation.
- The rotating panel will be constructed using 3/4" plywood for the given size (120") it will weigh approximately 20 lb.



CHILDREN'S MUSEUM	
DATE SUBMITTED:	1
REVISED:	
TEAM NAME:	

# Concept Generation

The concept generation conducted by the team began with general brainstorming on a variety of topics and continued with research and sketches of possible solutions.

## Brainstorming

### 1. Methods of Power Transmission

- a. Direct Mechanical (i.e gears and sprockets)
- b. Pulleys and Ropes (i.e. wire cable or stranded rope)
- c. Axle and Gearbox
- d. Linear Actuators
- e. Linkages or Levers
- f. Pneumatics (i.e. passive versus active, gas strut/powering system)
- g. Hydraulic
- h. Ratcheting Mechanisms
- i. Oscillating Mechanisms
- j. Electric Motor with Control System
- k. Cams/Rotating Plates
- l. Animal Power (i.e. horses, hamsters, dogs)

### 2. Handle Ideas

- a. Pivoting versus Rotating versus Sliding
- b. Shaft design (i.e. spherical, t-handle)
- c. Locking and release mechanisms
- d. Continual/proportional versus binary

### 3. Safety

- a. Braking/Locking Mechanisms
- b. FlyWheel (absorption of excess energy)
- c. Limit Switches, E-Stops, Force Sensors
- d. Variable Friction Belts
- e. Governor/ Fluid Resistance (speed limiting device)
- f. Counterweights
- g. Springs (absorb shock)

### 4. Aesthetic and Interactive Features

- a. Durability and safety are most important
- b. Visibility of mechanisms to operators for interest and safety
- c. Lighting component (i.e. LED's to indicate function or position)
- d. Sound component

### 5. Similar Mechanisms

- a. Hinging doors and ramps, wheelchair ramps, drawbridges, winches

## Research

Research was conducted on the topics listed below. For more, see [Appendix B: Research](#).

### 1. The Game *Guess Who?*



### 2. Ratcheting Mechanisms

- a. Include a wheel with teeth cut out, a shaft that the gear can turn on, and a pawl that follows the teeth in the wheel that maintains contact using a spring or lever system (ex. Socket wrench, clock, carjack, turnstile)
- b. A limitation with ratchets is that it can only provide torque in one direction but a changeover lever can allow the ratchet to provide torque in the opposite direction

### 3. Torque Limiters

- a. Ensure that torque based forces will not exceed a certain point by slipping once the torque is overloaded to prevent damage to your equipment by disconnecting the drive shaft from other components

### 4. Brake Winches

- a. Hold the load when the operator lets go of the crank
- b. One type is a worm drive where a worm meshes with a gear
- c. Second type is a brake clutch that aims to stop or slow the shaft from rotating without gears clashing to eliminate damage

### 5. Mechanical Advantage

- a. Ratio of output force to input force found in simple machines, like pulleys

### 6. Mechanical System Components

- a. Regardless of the drive source, power transmission is a great consideration
  - i. A gear is meant to mesh with another gear, making it a more simple option
  - ii. Sprockets use chain instead of meshing together which creates distance between the parts and can drive heavier parts
  - iii. Belts and pulleys are similar to sprockets and chain but are grooved instead of having teeth and prevent slippage because the belts are made from more flexible material, like rubber
  - iv. Shaft collars clamp around the shaft to facilitate proper movement



- v. Bushing or Bearings are often used to reduce friction between the shaft and housing to prevent damage

## 7. Mechanical Safety Mechanisms

- a. Torque overload prevention follows slip clutch functions
  - i. Friction types transmits torque but when the torque exceeds a threshold, friction disks will slip on each other to limit torque
  - ii. Shear pin types connect rotating components and a constant shear force is applied to the pins as torque is transmitted until there is an overload when the pins will break
- b. Limit Switches ensure safe operation even when there is a failure to define the limit of how far an object can travel (ex. Garage door opener)

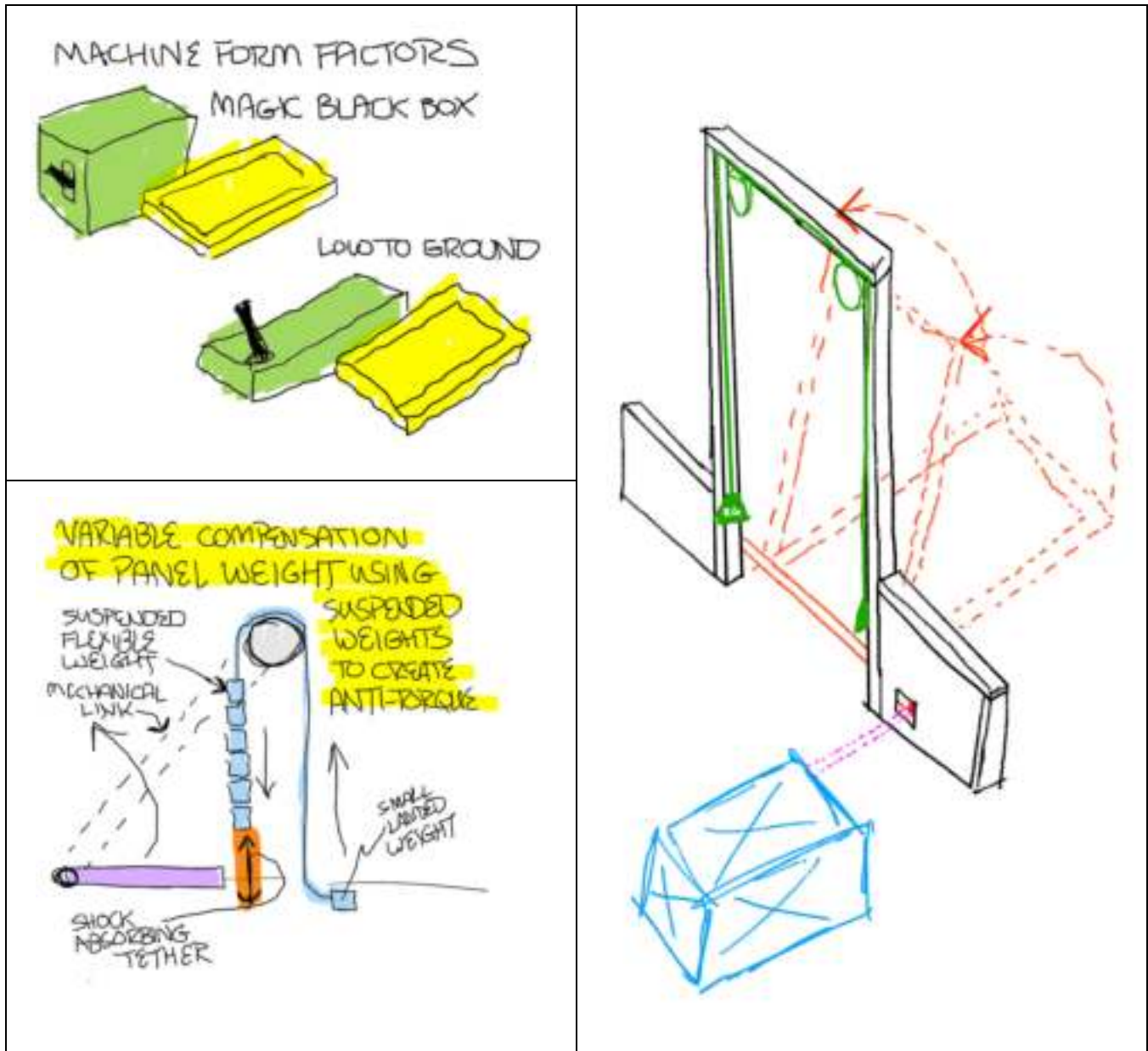
As part of its research, the team also sought clarification from Purdue and the SMDC regarding some questions that arose while evaluating the challenge and conducting brainstorming.

1. What size is the axle shaft key?
  - a. The key size is a standard  $\frac{1}{4}$ " x  $\frac{1}{4}$ "
2. The illustrated location/orientation of the operator in the drawing seems backwards to the way *Guess Who* is typically played. Is this intended?
  - a. The panel will move opposite how the pieces in the *Guess Who* game typically work so that the panel rotates up towards the operator, whereas the board game pieces are flipped up, away from the game player.
3. How much time will be allotted to set up the machine and can it be attached to the floor?
  - a. The panel will be free floating on the ground, but we have the ability to screw our materials to the ground as long as it is within our allotted 25 minutes to set up, troubleshoot, demonstrate for judges, and strike
4. Do the movement of the handle and the panel have to be directly proportional, or can the handle operate more like a switch or a trigger?
  - a. Handle movement should be directly proportional to the panel movement



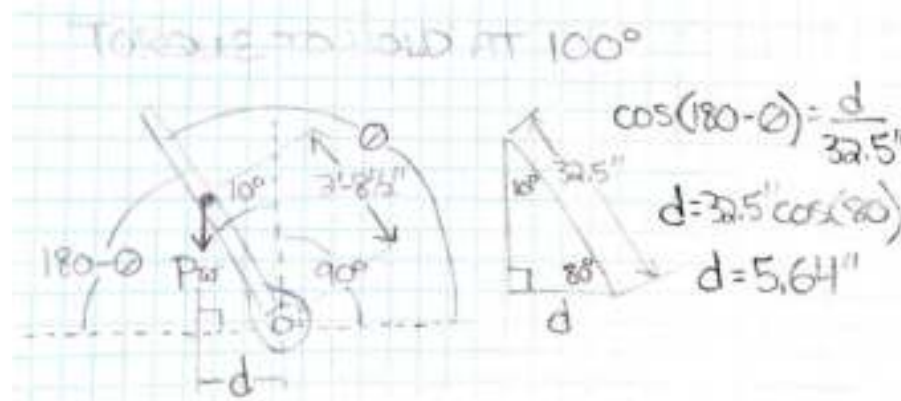
## Concept Sketches

The next step in concept generation was to sketch out ideas for the overall form factor of the machine, different methods of power transmission, etc. For a complete illustration of concept sketches refer to [Appendix A: Concept Sketches](#).



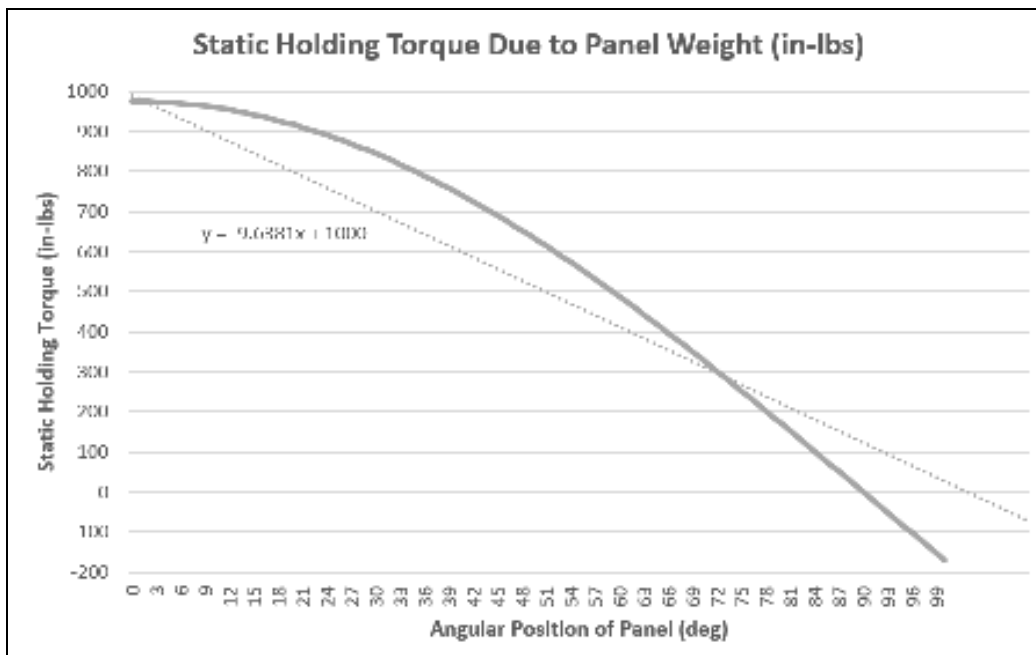
## Calculations

Some critical calculations were also part of the team's concept generation process. The first calculation of interest was the static torque required to hold the weight of the plywood panel at a given angle in its travel. For all the steps of this calculation, see [Appendix C: Calculations](#).



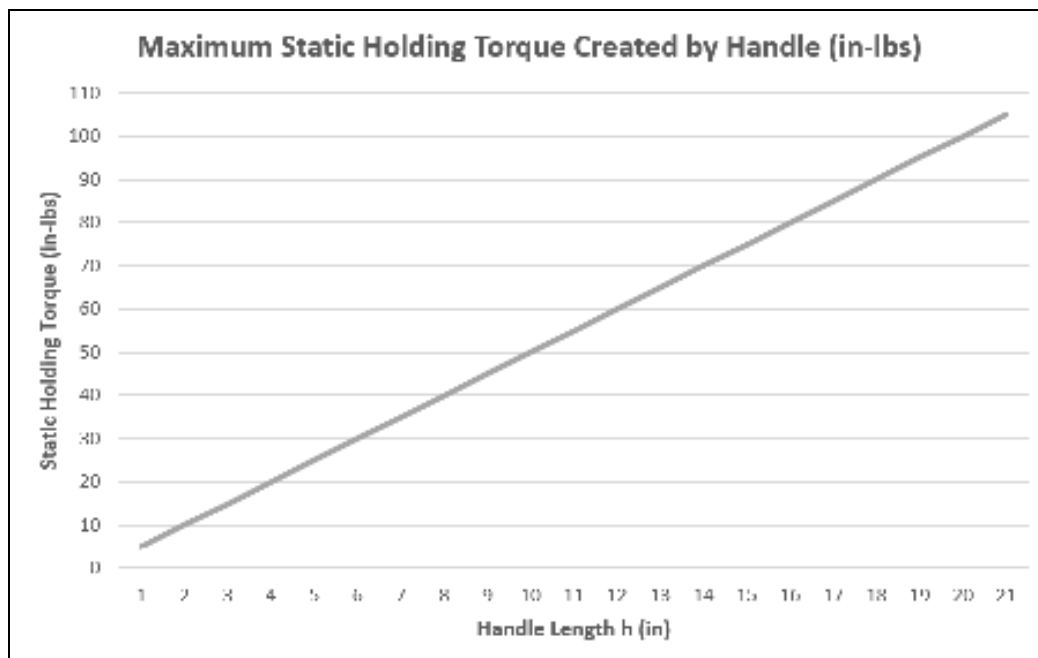
Calculation of Torque Required to Hold Panel at 100°

The static holding torque is at a maximum when the lever arm created by the panel's weight about its pivot is longest, which occurs in the horizontal orientation of the panel at 0 degrees. As the panel is raised, the lever arm between the pivot and the *line of action* of the panel's self weight shrinks to a minimum when the panel is vertical at 90 degrees. Beyond 90 degrees, the lever arm again increases in length increasing the required static holding torque.



The result of the previous calculation is that approximately 975 in-lbs of torque is required to hold the panel in the horizontal orientation. Note that even if the panel is resting on a stop or support of some kind, this is still the minimum torque required to begin lifting the panel.

Also of interest to the team was the maximum torque that could be generated by different lengths of the operator handle. Assuming the length of a simple lever handle could not exceed 21 inches, the maximum allowable 5 pounds of input force could generate a maximum torque of about 105 in-lbs. For round hand wheels limited to 8 inches in diameter due to the handle displacement limitation, the maximum input torque would not exceed 20 in-lbs.



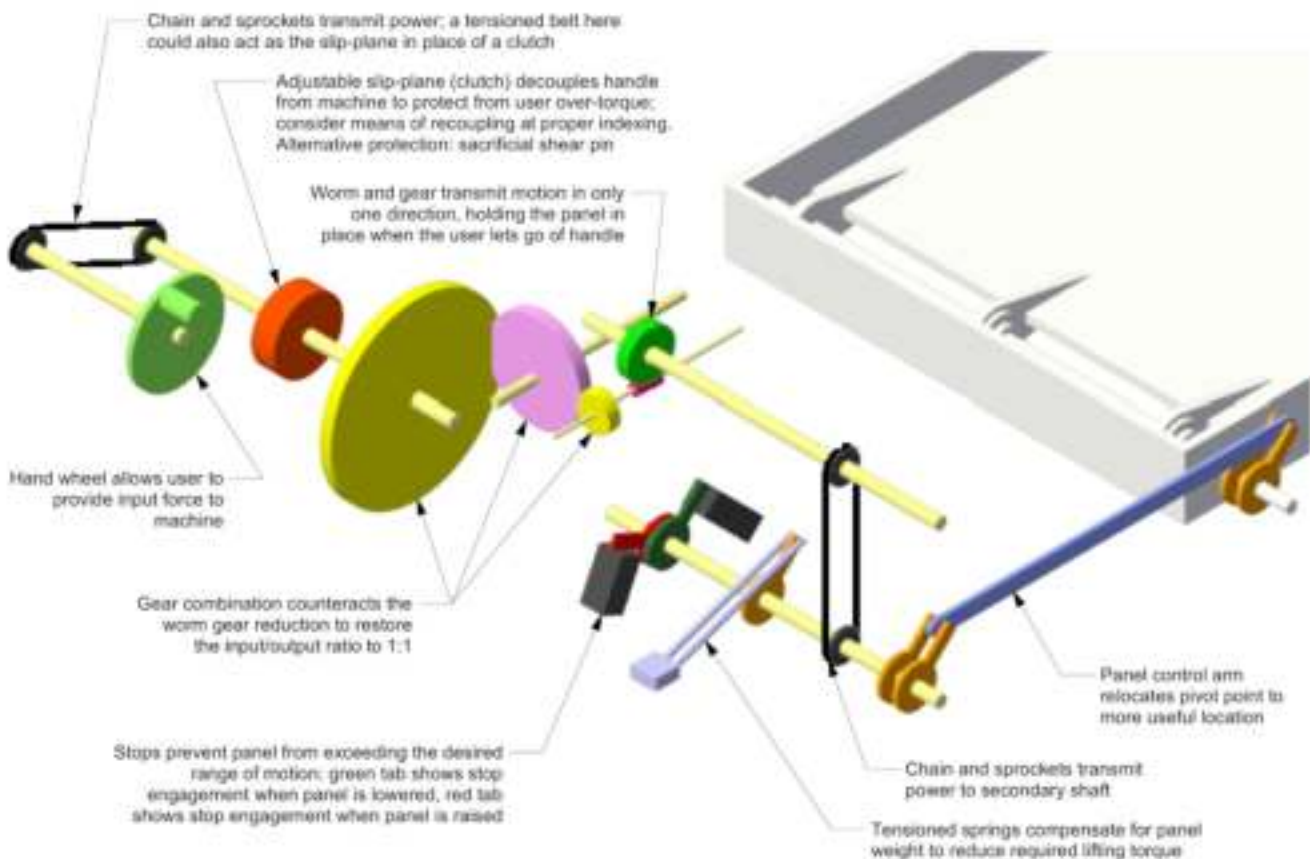
The handle calculations also enabled the team to review 1) the ratio between the available input torque and the required output torque to hold the panel horizontal and 2) the ratio between the available handle rotation and the required 100 degrees of panel rotation. Predictably, the longer the handle the more torque is available, but fewer degrees of rotation are achieved before the 8 inch displacement limit is reached. For graphs of this data, see [Appendix C: Calculations](#).

## Concept Selection

The selection of a concept to pursue was informed by the brainstorming and research the team conducted, the complexity of the problem presented in the challenge, as well as practical considerations regarding the resources available such as time, expertise, funding, etc. While a formal evaluation method such as a Pugh's Matrix was considered, ultimately the decision was made through general discussion.

## The Concept

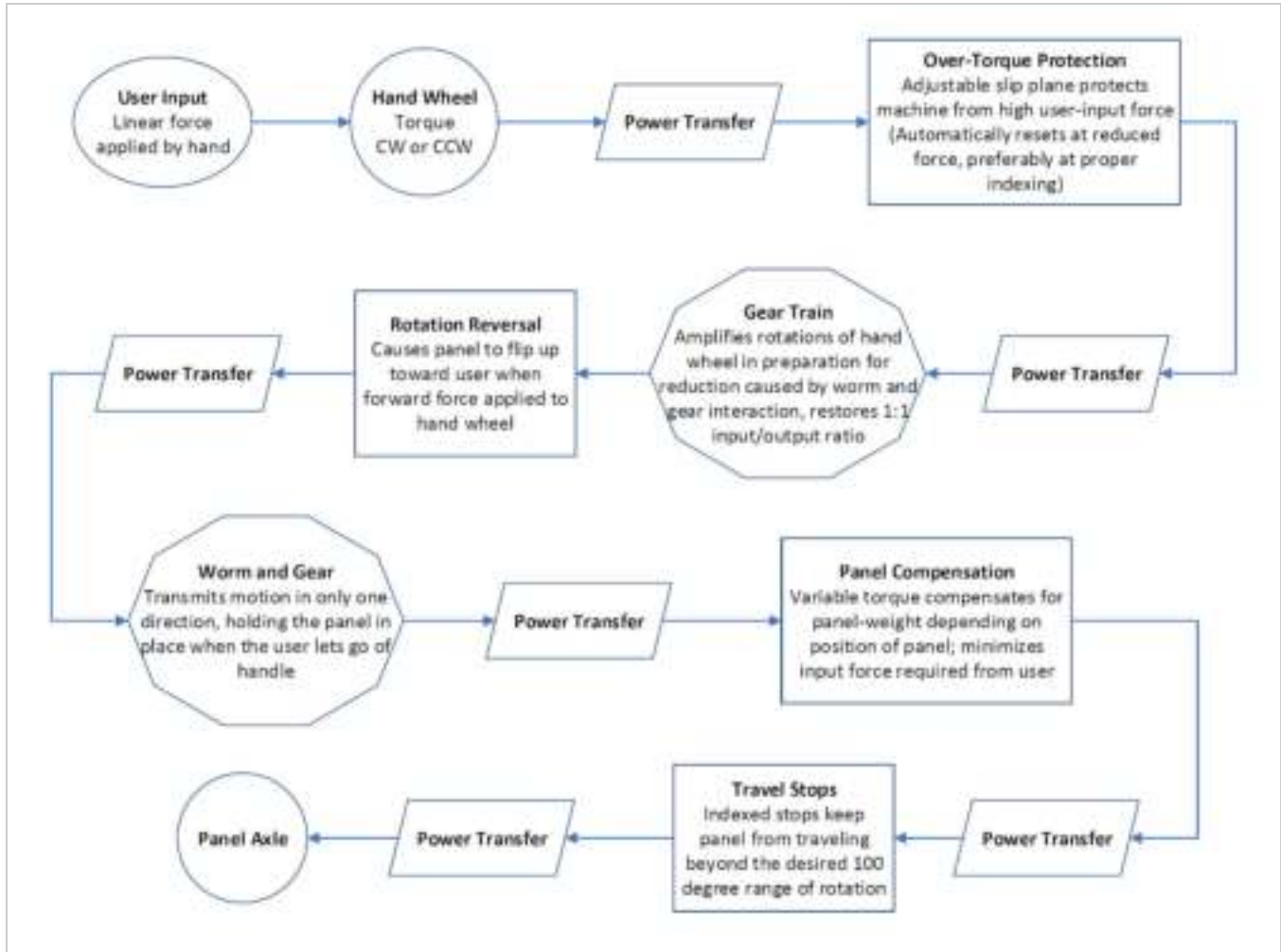
The team has chosen to design a machine which utilizes entirely mechanical components to directly harness the input force generated by the operator to achieve the desired rotation of the panel. Refer to the rough conceptual model below for details on the proposed power transmission methods and features. Note especially the use of a round hand wheel, an over-torque slip-plane, a worm and gear, and springs to compensate for the weight of the panel. Note that the actual machine will vary from this model in layout, component design, division of functions across components, etc.



*Rough Conceptual Model*

## Functional Decomposition

This flowchart illustrates the general functions of each power transmission step in the machine. The detailed design process will further breakdown and assign these general functions to individual components of the machine.



*Functional Decomposition*

## Assessment

### Pros:

- Does not require electricity or other consumable/rechargeable fuels or gasses
- Individual components are relatively simple (no electrical circuitry for example)
- The force and potential damage that can be caused by a human during testing and operation is less than by other methods of power transmission such as electric motors, pneumatics, hydraulics, etc
- Hand wheel can rotate continuously if decoupled during over-torquing; its position is less restricted than a lever-based handle
- Worm gear is a simple mechanical device capable of holding the panel in place if the operator lets go of the handle (it cannot be back-driven by the panel)
- Compensation of the panel weight using spring force can reduce the amount of input torque required from the user
- Achievable with the resources available to the team

### Cons:

- Direct mechanical coupling of the operator and the panel provides opportunity for an uncontrolled panel to harm the operator, or for the operator to damage the machine
- Sophisticated safety systems such as sensors, E-stops, etc are not available
- Tuning of the mechanical system to minimize friction losses, transmit torque, and compensate for the panel's weight while not exceeding the input force may be tricky
- Sudden decoupling of the hand wheel from the machine to protect it from over-torque could cause injury to the operator
- Compensation of the panel's weight can't be perfect over the full range of movement
- Worm gear significantly reduces the gearing ratio between the operator handle and the panel, which requires reversal elsewhere in the system to maintain the 1:1 proportional relationship between the handle and the panel



## Design Specification

From the challenge details and the team's brainstorming, research, and concept selection, the specific requirements that the team's machine must fulfill are listed below.

The designed machine shall:

1. Rotate a 3' x 5' panel (weighing roughly 30 pounds) 100 degrees from its horizontal resting position and back again via a 1" keyed shaft.
2. Accept mechanical input through an operating handle that sits 21 inches from the floor, travels no more than 8 inches, and requires no more than 5 pounds of force to be applied. The handle will be located/oriented so the panel rises up toward the operator while keeping the operator at a safe, appropriate distance from the panel.
3. Lift the panel when a pushing force is applied to the handle, and lower the panel when a pulling force is applied to the handle.
4. Have a form factor which does not grossly overshadow or upstage the panel and does not significantly inhibit the operator's view of the panel during and after operation.
5. Include compensation for the weight of the panel to minimize the input torque required from the operator and therefore also the stress put on many of the components.
6. Have a built-in safety function that holds the panel in position when the operator lets go of the handle, preventing unwanted or dangerous movement from feeding back into the handle and harming the operator.
7. Have a function that disengages the handle when high torque is applied by the user to protect the machine components from damage.
8. Include protective covers and features which prevent the operator's limbs, hair, clothing, or accessories from becoming entangled in the moving components of the machine.
9. Be free from sharp points, edges, or burrs which could cause injury if touched.
10. Include components and a supporting frame appropriate to resist all forces and stresses experienced during highly repetitive operation over the course of the museum's event.
11. Allow for setup and operation of the machine within 20 minutes.
12. Achieve all of the above functions through entirely mechanical means; electricity or consumable/rechargeable fuels or gasses will not be needed for operation.



## Next Steps

### Detailed Design

Below is the team's checklist for the detailed design process which will now commence.

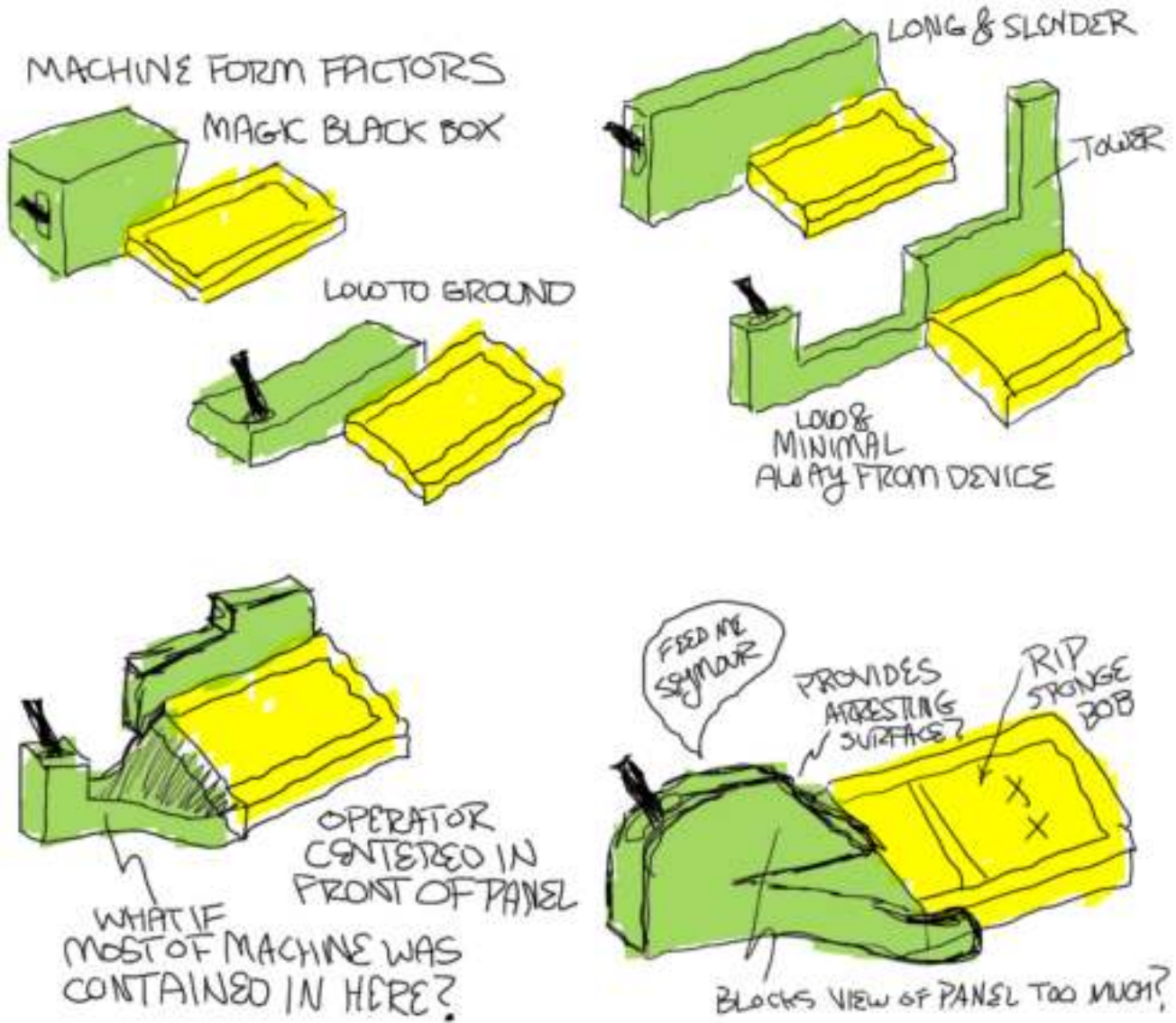
- Divide design responsibilities among team members
- Pull useful stock components from storage and old projects
- Collect useful scrap or surplus material
- Conduct calculations necessary for panel weight compensation
- Determine specialty components which may need to be purchased
- Conduct design of components and assemblies, leading to build drawings
- Conduct prototyping as necessary for key components

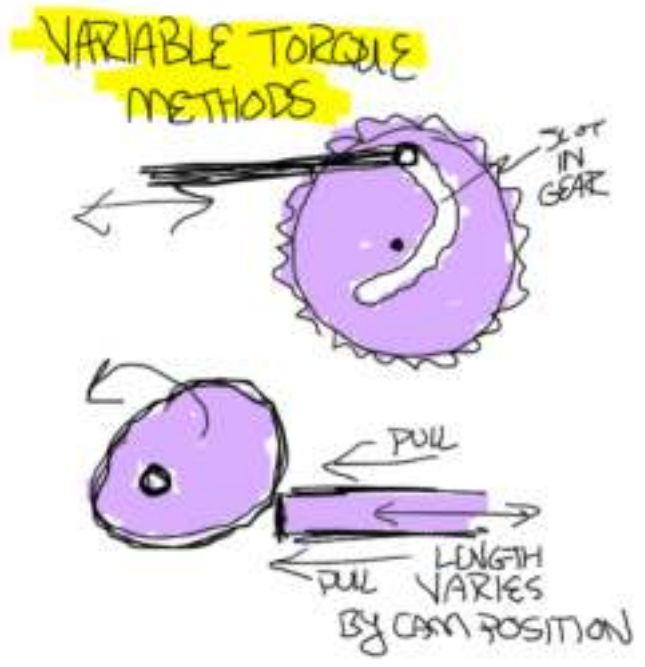
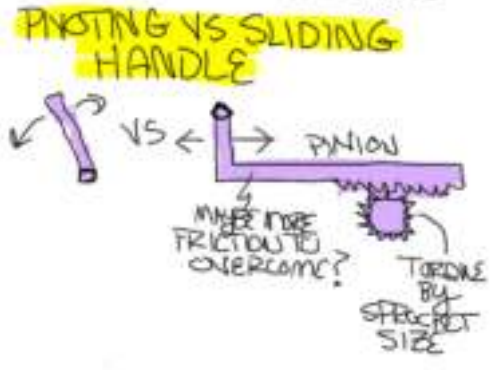
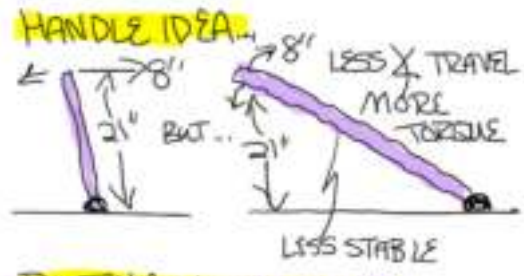
### Build and Testing

In conjunction with the build of the machine, the team will also construct a replica of the challenge panel so that accurate fitment and operational testing can be performed. A 3D model of the challenge panel has already been created in preparation for this task.

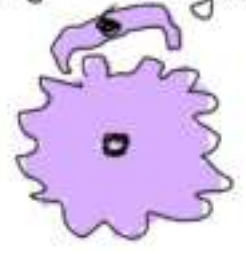
During build and testing, the team will prepare for a smooth installation process at the competition. At the completion of this work, the team will document its design process, build progress, testing results, and conclusions in a final report which will accompany the team to the competition.

# Appendix A: Concept Sketches



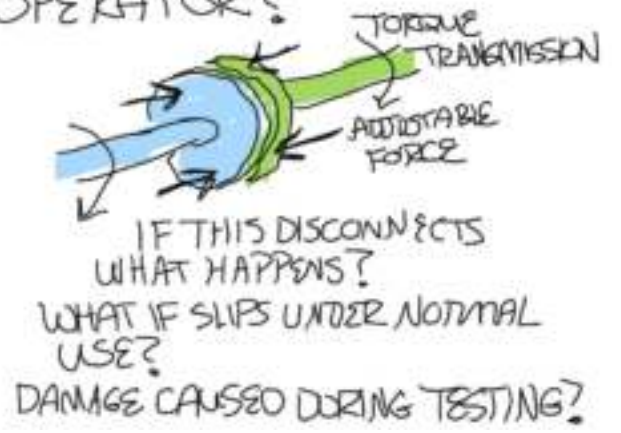


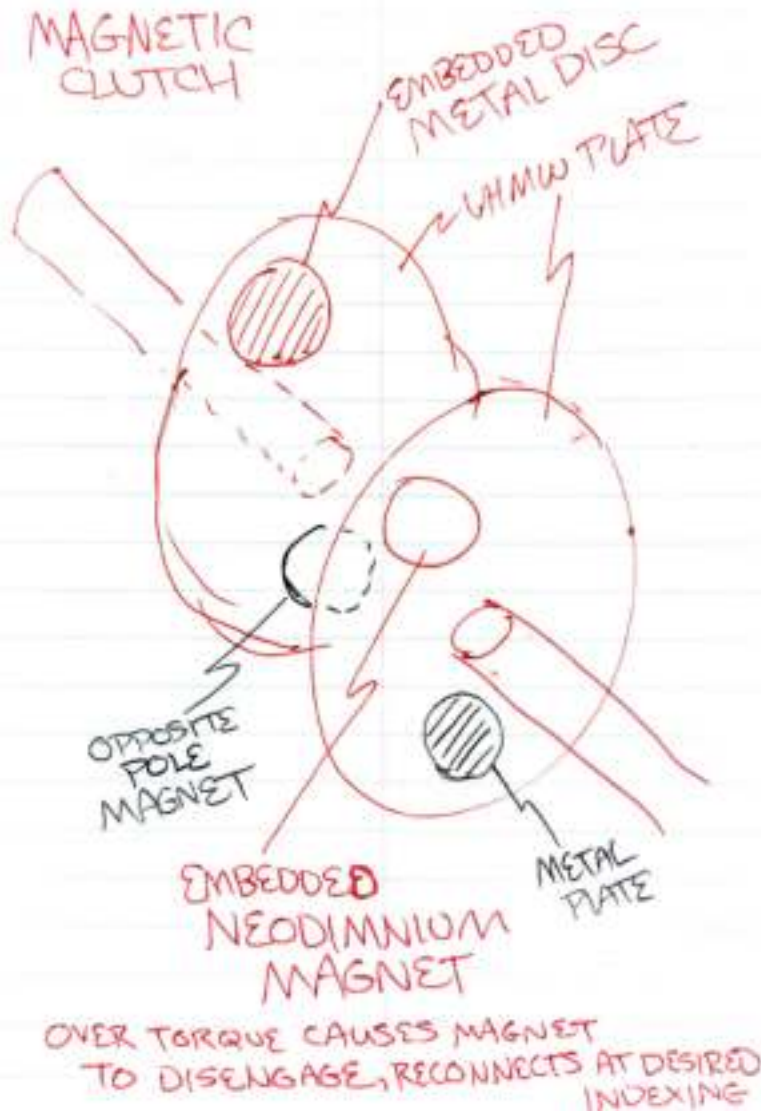
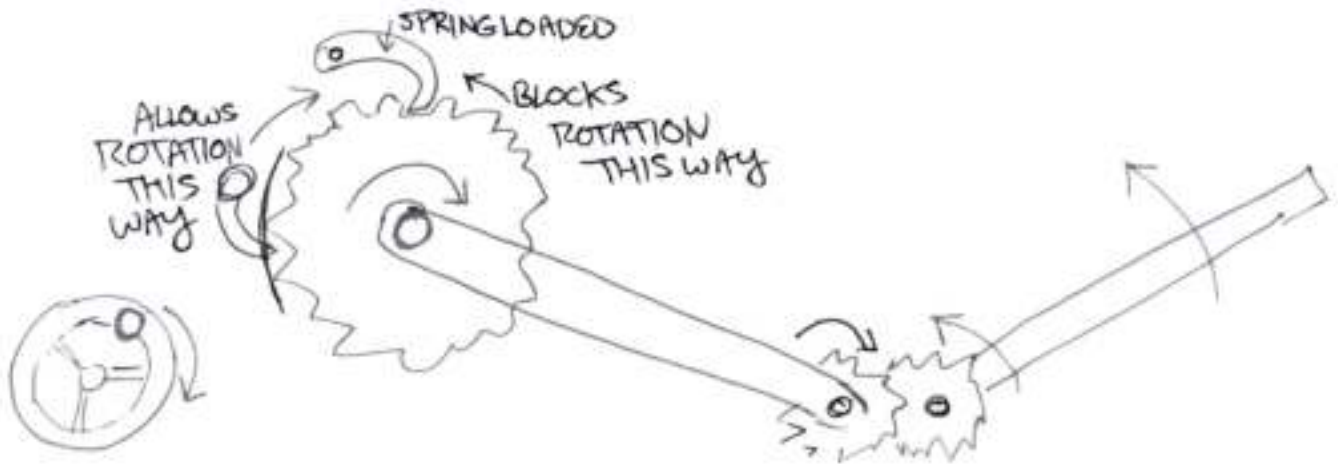
**REVERSING RATCHET MECHANISM THAT CHANGES DIRECTION AT EACH TRAVEL EXTENT? OR ANYWHERE?**



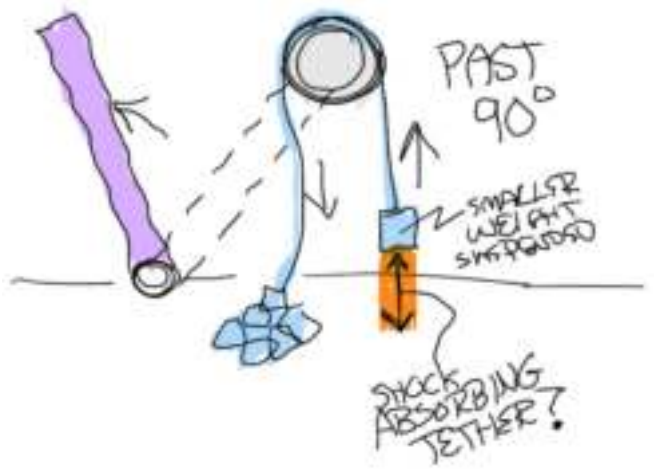
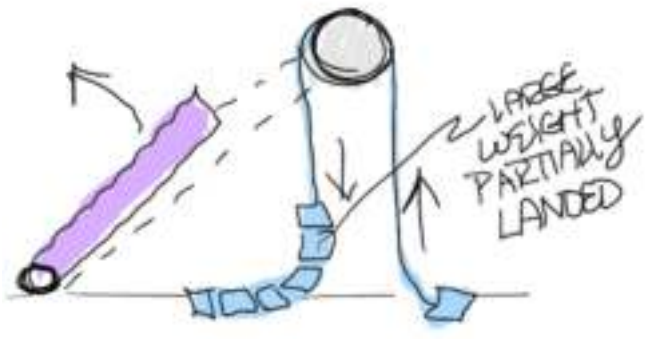
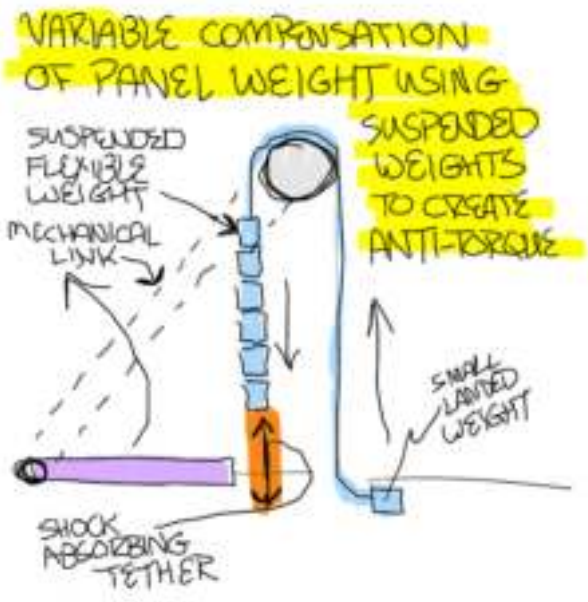
NOTE: IF THE PANEL WEIGHT IS COMPENSATED FOR THE AMOUNT OF INPUT FROM OPERATOR COULD BE VERY LOW

**ADJUSTABLE CLUTCH INTERFACE TO PROTECT MACHINE FROM FORCEFUL OPERATOR?**

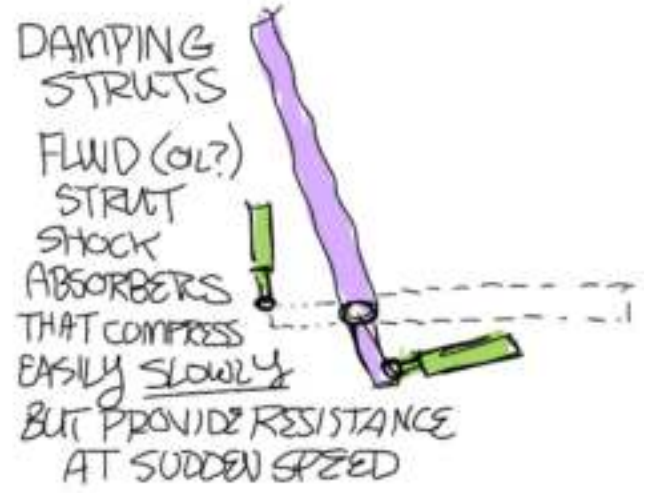


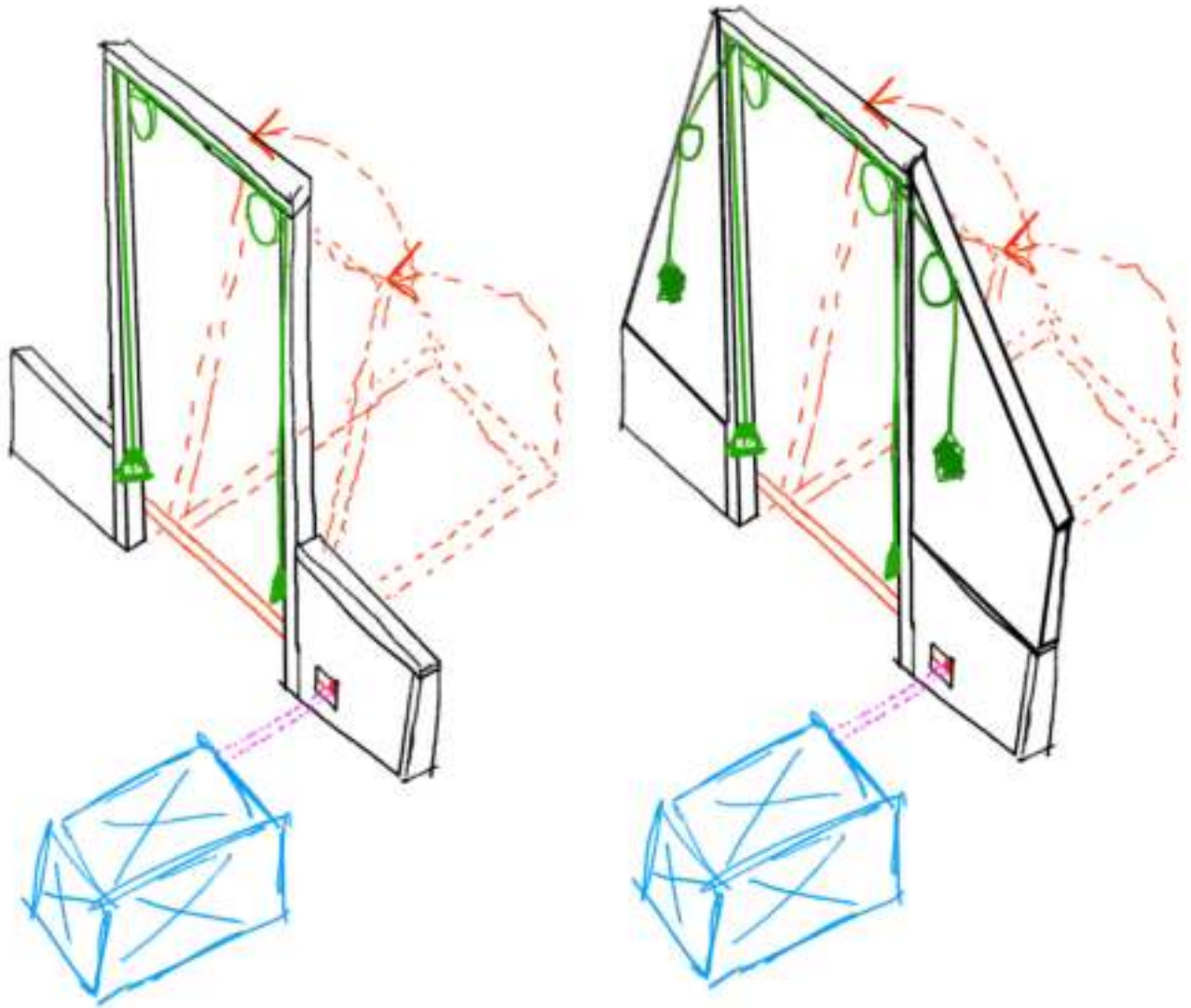






POTENTIAL FOR SMALL IMBALANCE TO RUN AWAY?  
SHOCK ABSORBING MECHANISM?  
BRAKING MECHANISM?



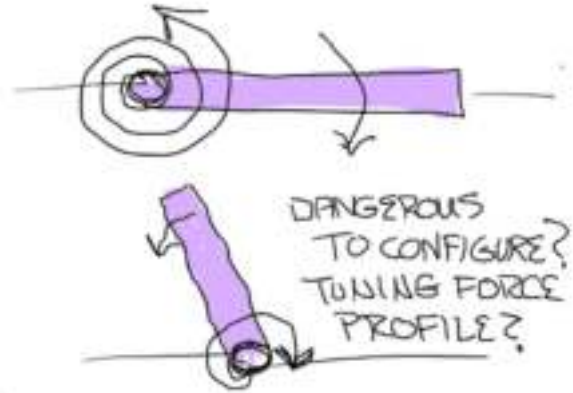


**OTHER ADJUSTABLE SUP MECHANISM...**

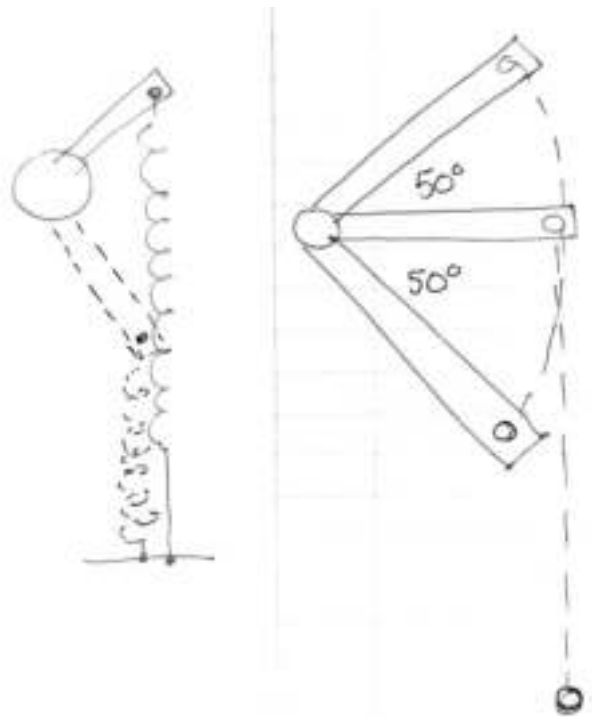
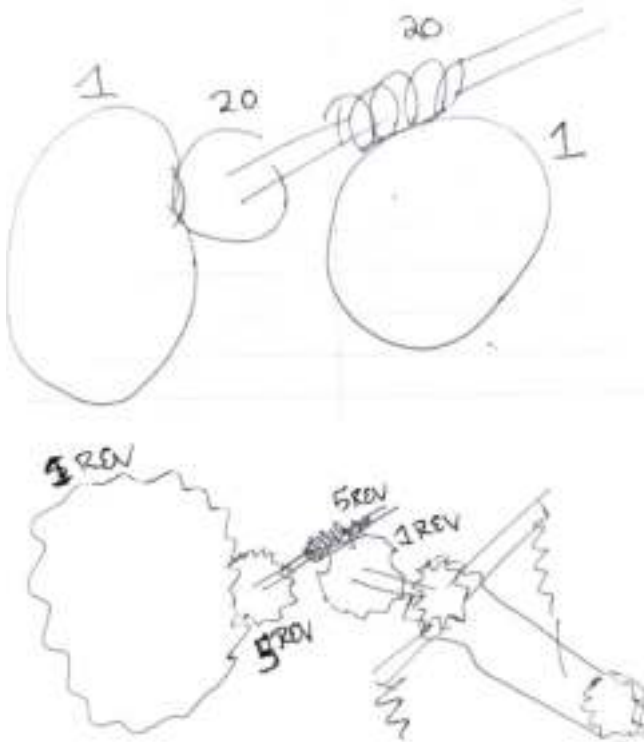


COMBINE THIS WITH A RATCHET MECHANISM TO HOLD THE LOAD?

**USE OF TORSION SPRINGS OR OTHER SPRINGS**



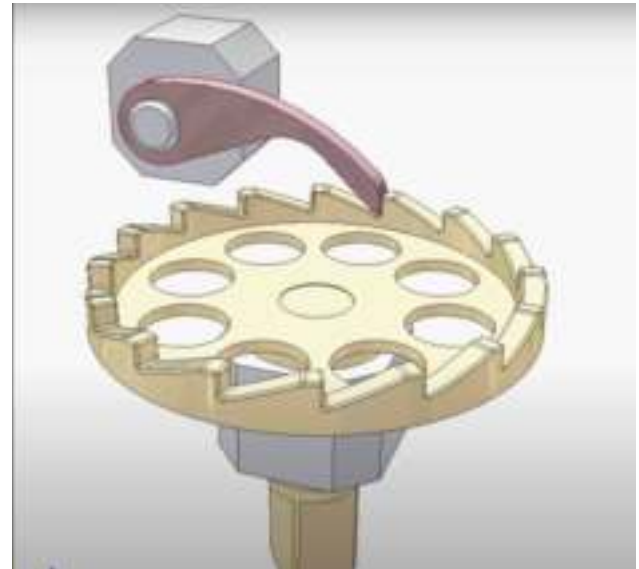
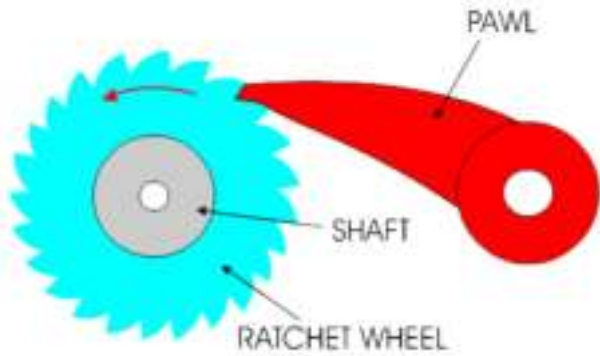
HOW MUCH TIME DO WE HAVE TO SET UP AND TUNE?





## Appendix B: Research

### Ratcheting Mechanisms



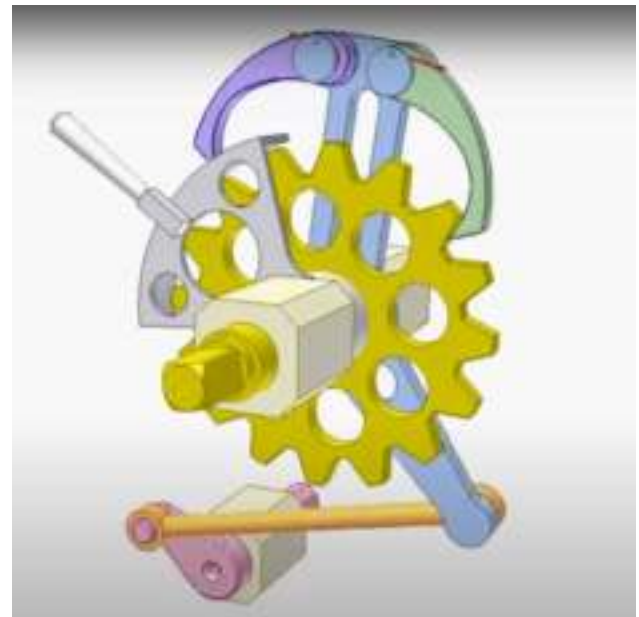
*Anti-Reverse Ratchet Mechanism on [YouTube](#)*



*MetMo Ratcheting Driver on [Kickstarter](#)*

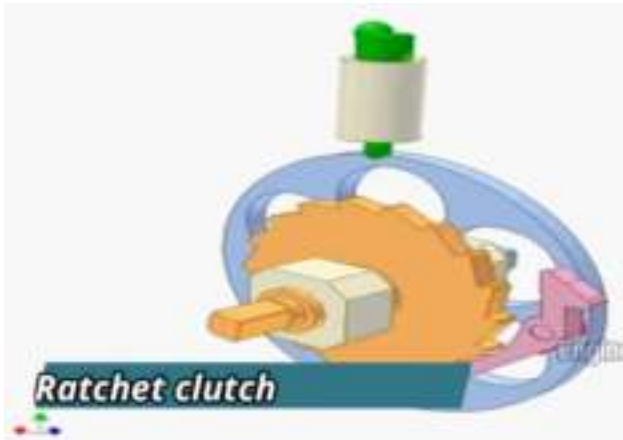


*Reversible Ratchets on Pulling Winches from [Dutton-Lainson](#)*

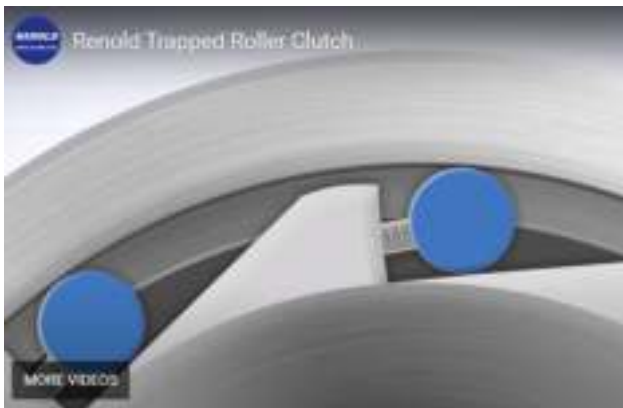


*Two-Way Ratchet Mechanism on [YouTube](#)*

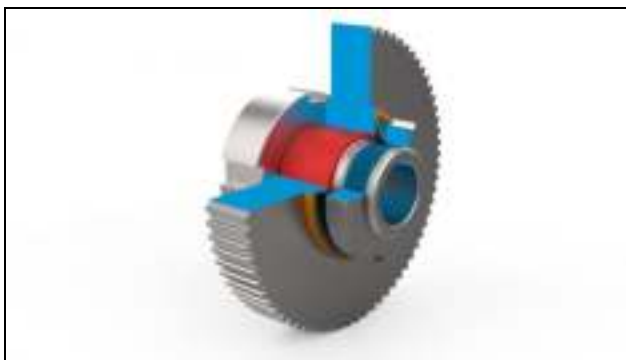
## Clutch and Torque Limiting Devices



Ratchet Clutch on [YouTube](#)



Trapped Roller Clutch by [Renold](#)



The Mechanical Torque Limiter  
[Saint-Gobain.com](#)

Learn to Let Go with Torque Limiters  
[MachineDesign.com](#)

MINI MAGNETIC CLUTCH.



3D-Printable Mini Magnetic Clutch from  
[cults3d.com](#)

## Sprockets



Types of Sprockets from [ustsubaki.com](#)

Basic Concepts of Mechanical Systems  
[acastronovo.com](#)

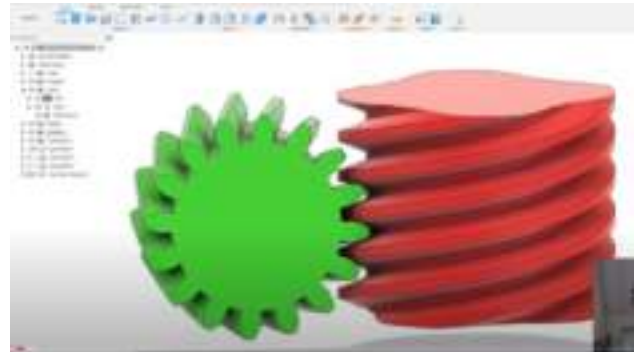
## Brake Winches and Worm Gears



Brake Winches by [Dutton-Lainson](#)



3D Printed High Torque Gearbox from [Thingiverse](#)



How to Model Worm Gears in Fusion360 from [YouTube](#)

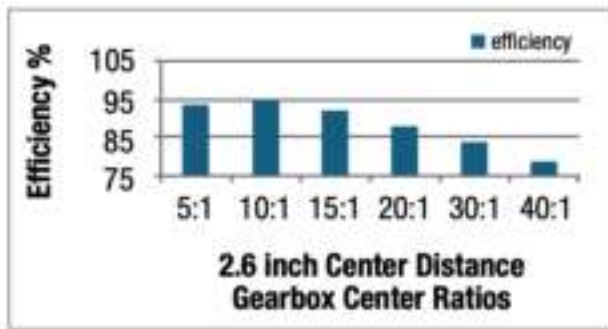


How Does a Brake Work on a Hand Winch by [Gebuwin](#)



3D Printed Worm Gearbox from [YouTube](#)





What determines worm gear efficiency, and is it really that low? From [MotionControlTips.com](http://MotionControlTips.com)



Worm Gear from [Thingiverse](http://Thingiverse)

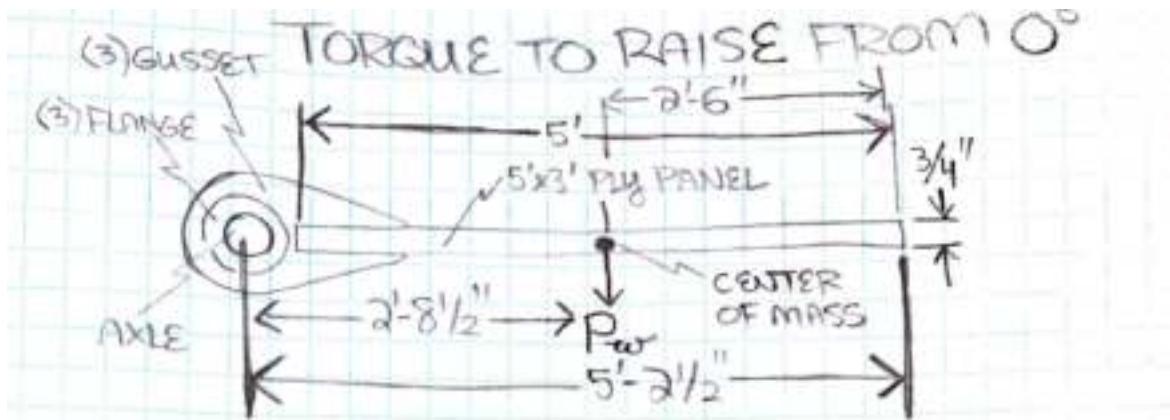


5:1 Worm Gear Speed Reducer from [OMC-StepperOnline.com](http://OMC-StepperOnline.com)



LiftMaster Worm and Gear Garage Door Opener Kit from [LiftMaster](http://LiftMaster)

## Appendix C: Calculations



TORQUE REQUIRED TO LIFT PANEL FROM HORIZONTAL POSITION?  $T_0$

WILL NEGLECT STATIC FRICTION BETWEEN METAL AXLE AND WOOD FRAME

WILL NEGLECT SELF-WEIGHT OF GUSSETS

$T_0 > T_{iw}$  TORQUE TO LIFT MUST BE GREATER THAN TORQUE CREATED BY SELF-WEIGHT OF PANEL

$$T_0 = P_w * d \text{ WHERE } d = 32.5''$$

PER DRAWING,  $P_w \approx 28 \text{ lbs}$

PER STRUCTURAL DESIGN FOR THE STAGE APPNDX F, APPROX WEIGHT OF 3/4 PLY IS 2.2 PSF

$$P_w = \frac{2.2 \text{ lbs}}{\text{ft}^2} (3\text{ft}) (5\text{ft}) = 33 \text{ lbs}$$

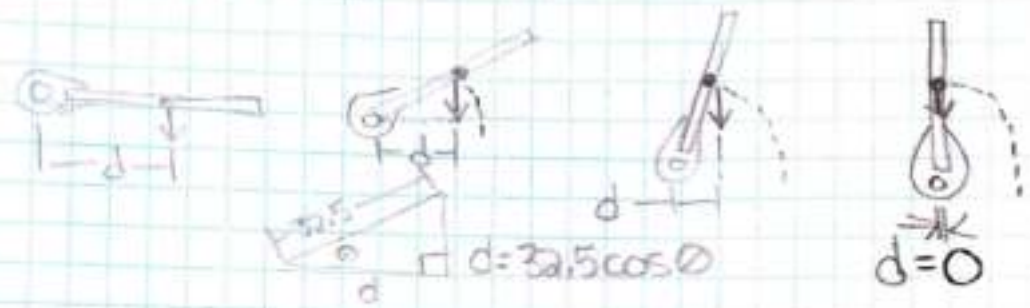
SPLITTING THE DIFFERENCE...

$$T_0 > 30 \text{ lbs} (32.5 \text{ in})$$

$$T_0 > 975 \text{ in lbs OR } 81.25 \text{ ft lbs}$$

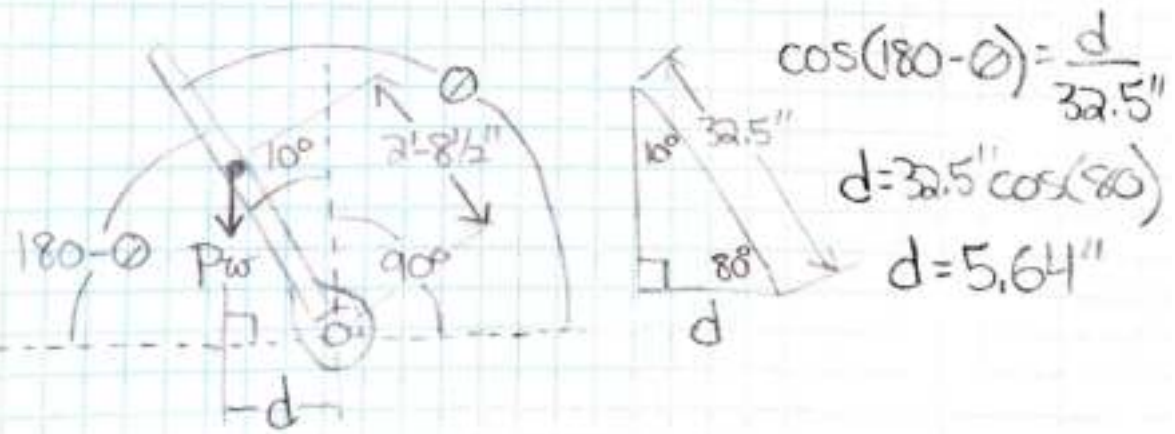
NOTE: THIS TORQUE WOULD BE REQUIRED AT REST TO HOLD THE PANEL HORIZONTALLY IF NOT RESTING ON ANYTHING

NOTE: THE LIFTING TORQUE DECREASES AS THE PANEL IS RAISED DUE TO THE SHRINKING  $d$  VALUE



NOTE: THE ACTUAL TORQUE REQUIRED TO BEGIN LIFTING WOULD ALSO DEPEND ON ANY RESISTANCES WITHIN THE MACHINE

TORQUE TO HOLD AT  $100^\circ$



$$T_{100} = P_w d = 30 \text{ lbs} (5.64 \text{ in})$$

$$T_{100} = 169.2 \text{ in lbs OR } 14.1 \text{ ft lbs}$$

HOLDING TORQUE AT ANY ANGLE  $\theta$

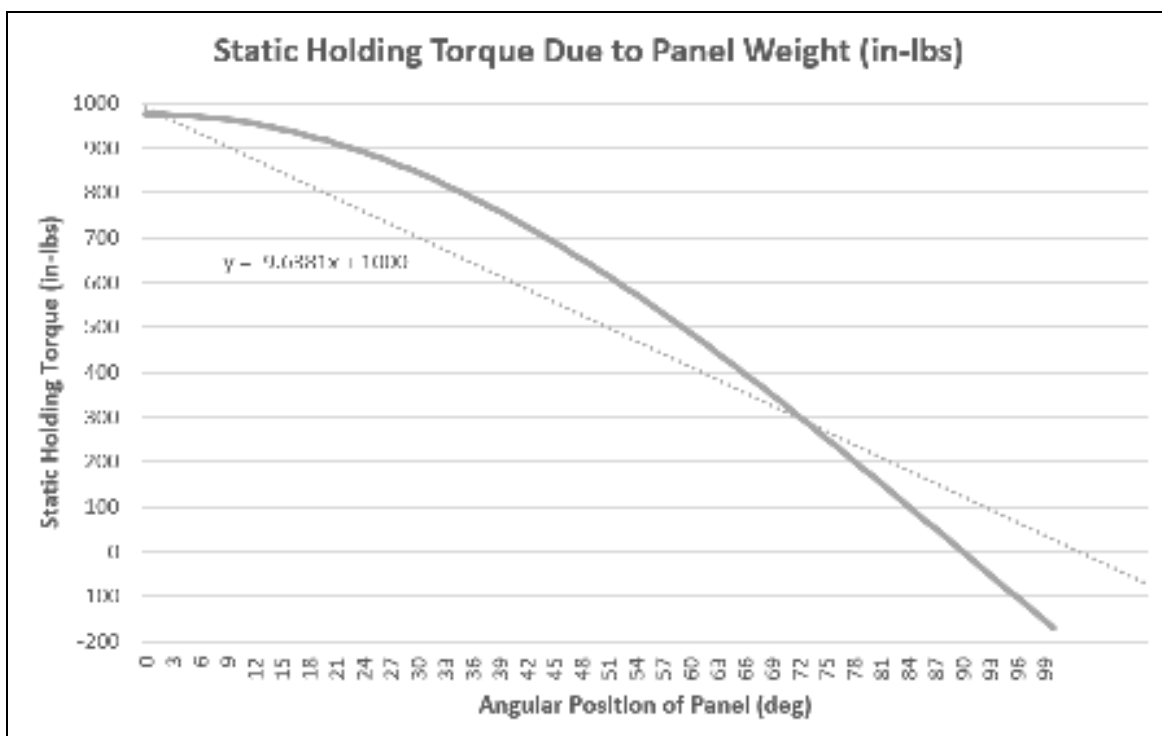
$$\text{IF } \theta \leq 90^\circ \quad d = 32.5'' \cos(\theta)$$

$$\text{IF } \theta > 90^\circ \quad d = 32.5'' \cos(180 - \theta)$$

$$T_\theta = d P_w \quad (\text{in-lbs})$$

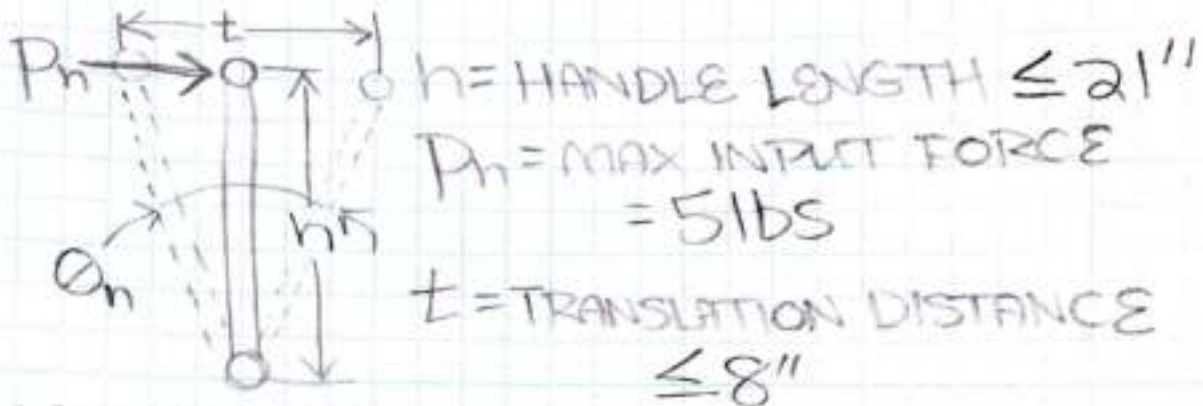
$$T_\theta = d P_w / 12 \quad (\text{\#lbs})$$

NOTE: THE PREVIOUS IS ALL STATIC HOLDING TORQUE REQUIRED; IT DOES NOT ACCOUNT FOR DYNAMIC EFFECTS OF THE PANEL MOVING AND POSSIBLE ASSOCIATED SHOCK LOADING CAUSED BY SUDDEN ACCELERATION/DECELERATION DUE TO THE OPERATOR OR ACTUATING MECHANISM, OR THE PANEL FREE-FALLING.





## LEVER/HANDLE TORQUE INPUT

LEVER DESIGN  $\theta_h = \text{HANDLE ROTATION ANGLE}$ 

$$T_h = \text{TORQUE DUE TO HANDLE} = P_h * h$$

ASSUME THAT  $P_h$  TRAVELS TANGENTIALLY WITH LEVER:

WHAT ANGULAR MOTION RESULTS FOR DIFFERENT HANDLE LENGTHS?



$$\sin\left(\frac{\theta_h}{2}\right) = \frac{t/2}{h} = \frac{t}{2h}$$

$$\frac{\theta_h}{2} = \sin^{-1}\left(\frac{t}{2h}\right)$$

$$\theta_h = 2 \sin^{-1}\left(\frac{t}{2h}\right)$$

